Math 10B with Professor Stankova
Quiz 9; Tuesday, 4/2/2019
Section #203; Time: 11 AM
GSI name: Roy Zhao **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** The CDF is the derivative of the PDF.

Solution: It is the opposite, the PDF is the derivative of the CDF.

2. True **FALSE** Suppose that $f(x) = -1$ for $-1 \leq x < 0$ and $f(x) = 1$ for $0 \leq x \leq 2$ and 0 everywhere else. Then f is a PDF.

Solution: This is false since $f(-1) = -1$ which is negative and PDFs cannot be negative.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (2 points) Suppose that $f(x) = Cx^3$ for $0 \leq x \leq 2$ and $f(x) = 0$ for all other x for some constant C. If f is a PDF, then find C.

Solution: Since f is a PDF, we require that

$$\int_{-\infty}^{\infty} f(x)dx = \int_0^2 Cx^3 = 1.$$

This integral is

$$\int_0^2 Cx^3 = \frac{Cx^4}{4}|_0^2 = 4C = 1.$$

Therefore $C = \frac{1}{4}$.

(b) (4 points) Find the CDF of f from above. (Hint: the CDF will be piecewise)

Solution: For $x \leq 0$, then the CDF is 0 because the PDF is 0 there. Then for $0 \leq x \leq 2$, we have that the CDF is

$$F(x) = \int_{-\infty}^{x} f(t)dt = \int_0^x f(t)dt = \frac{t^4}{16}|_0^x = \frac{x^4}{16}.$$

So

$$F(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x^4}{16} & 0 \leq x \leq 2 \\ 1 & x \geq 2. \end{cases}$$

(c) (4 points) Find the mean and median of the PDF f from above.

Solution: The mean is

$$\int_0^2 x\frac{x^3}{4}dx = \int_0^2 \frac{x^4}{4} = \frac{x^5}{20}|_0^2 = \frac{32}{20} = \frac{8}{5}.$$

The median is when the CDF is $\frac{1}{2}$ which is when $\frac{x^4}{16} = \frac{1}{2}$ or at $x = \sqrt[4]{8}$.